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Cite as: AIP Conference Proceedings 2114, 060011 (2019); <https://doi.org/10.1063/1.5112482>
Published Online: 26 June 2019

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Performance Measurement on Lease Equipment with Overall Equipment Effectiveness

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Abstract. The paper proposes equipment maintenance strategies using age reduction methods. The chosen maintenance strategy is the optimal preventive maintenance (PM) policy, which is expected to minimize total costs. In addition, the proposed strategy is expected to improve equipment performance as measured by the Overall Equipment Effectiveness (OEE). PM is done to reduce failure and avoid penalties as a result of failure. Meanwhile, Corrective Maintenance (CM) is carried out by the lessor (equipment owner) to correct the failure with minimal repairs in the lease period. In the present study, an efficient algorithm was developed to obtain an optimal PM policy and a closed form solution was obtained for case where the lifetime distribution of the equipment was Weibull. The total cost expected to use the optimal PM policy under the proposed maintenance scheme was then compared with the performance of other policies under various maintenance schemes through numerical examples.

INTRODUCTION

The Overall Equipment Effectiveness (OEE) provides an overview of the conditions of an engine that are determined by the level of availability, level of performance, and level of quality. The availability level measures the effectiveness of maintenance of production equipment under on-going production conditions. The performance level measures the level of the effectiveness of production equipment. Meanwhile, the level of quality measures the effectiveness of manufacturing processes to eliminate scrap, rework, and loss of results[1]. The Japan Institute of Plant Maintenance sets a minimum standard for OEE value of 85%[2]. Manufacture industries have been used the OEE concept to measure the effectiveness of equipment. It serves to ensure that the machine is functioning properly. Therefore, the lessor carries out maintenance activities as a responsibility to the tenant. This is what distinguishes OEE in the manufacturing industry with the leased equipment. OEE application for manufacturing industries reveals possible losses in the production process while OEE leased equipment discloses losses based on service factors from the lessor. Methods based on service factors on the interaction between equipment parameters, service processes and outputs are very important for the equipment industry being leased.

Despite of its importance, maintenance requires a relatively high cost. Mobley [3] claimed that maintenance activities cost between 15 and 40% (on average 28%) of total production costs. Eti and Ogaji [4] argued that maintenance costs accounted for 40% of the operating budget. Previous studies suggested that the maintenance costs are approximately 25% of the total operating costs[5][6]. A high maintenance fee is a substantial problem that must be solved immediately. In addition, maintenance activities require special skills to sustain and improve the machine.

Nevertheless, maintenance is considered inefficient for industries with certain equipment. Therefore, there is a disposition to rent rather than buy equipment[7]. This issue has been discussed several times [8-16], including the

threshold of machine failure, the optimal PM interval, the duration of rent and the profits. The main criterion in determining policy is by minimizing costs. Jaturnonatee [15] developed a PM sequential scheme with minimal improvement in new machines in which several parameters, including the number of PM actions, PM degrees, and optimal time intervals, are determined. Jaturnonatee [15] also modeled PM actions with a failure rate reduction (FRRM) method, but the developed model is not easy to implement.

Based on the existing methods, several studies have used the failure rate reduction/FRRM PM methods [15-17]. FRRM reduces the rate of equipment failure by determining a fixed amount or an amount equivalent to the current failure rate after action [18]. In addition, other studies also used the age reduction methods/ARM [8,10,19]. ARM is the age of equipment that is returned younger than present age with a fixed amount after each PM action [18]. The majority of researchers used total maintenance cost as optimal decisions [8,9,12,14-16,20]. Moreover, to minimize costs, Yeh and Chang [17] used failure rate as thresholds, while Schutz and Rezg [10] used reliability and Mabrouk et al. [9] used downtime approach. Briefly, maintenance activities do not only affect total maintenance cost but also downtime, failure rates, reliability, and also the Overall Equipment Effectiveness (OEE). The present study assumes that distribution failures follow the Weibull distribution, and repairs are made with minimal repairs if damage occurs, hence maintenance concepts with preventive maintenance is proposed to reduce failure rate during the rental period.

MATHEMATICS FORMULATION

According to Yeh et al. (2009), the ability level, $f(t)$, is feasible to increase the function (slump of equipment) from time to time (t) with $f(0) = 0$, in which during the lease period, work that fails by using minimal repairs by the lessor with fixed repair costs (Cm). It is an attempt for having minimal improvements and operational equipment. However, failure remains the same as it is performed similarly just before failure. The assumption is that each minimal improvement requires a random time (Tm), which is the general cumulative distribution function (G). In addition, if the time that has been changed exceeds the predetermined value (τ), then there is a $C\tau$ per unit time penalty for the lessor. Hence, the expected total corrective cost to the lessor at each failure is $Cm + C\tau \int_{\tau}^{\infty} G(t) dt$.

To reduce the number of failure, the lessor can perform the PM step in the lease period. After doing the PM action at the time of t_i , increase the equipment by a fixed amount $\delta \geq 0$, where $0 < t_1 < t_2 < \dots < T_n < L$. In practice, the cost of PM action is a non-negative and nondecreasing function of maintenance degree $\delta \geq 0$. In the present study, an example where PM costs for $Cpm(\delta)$ increase linearly with the degree of maintenance δ is presented, which is $Cpm(\delta) = a + b\delta$, where $a > 0$ and $b \geq 0$ are fixed costs and variable costs for each PM action, respectively. Furthermore, it is assumed that the time needed to make minimal repairs and PM actions is significantly different compared to the leased period and, negatively, can be ignored.

Without PM action, the equipment failure process is a Non-homogeneous Poisson Process (NHPP) with intensity $f(t)$, because minimal improvement Nakagawa [21-22] corrects failure. As a result, the expected number of failures in the interval $[0, t]$ is $F(t) = \int_0^t f(u) du$. When the PM action is carried out, the equipment failure process in each interval $[t_i, t_{i+1}]$ is still a NHPP. After the action with PM, however, the failure intensity becomes $f(t_i) - i\delta \geq 0$, for all $i = 1, 2, \dots, N$. According to NHPP, the expected amount of failure in the lease period under the proposed PM scheme is expressed as follows:

$$\Lambda = \Lambda(n, \delta, t) = \sum_{i=0}^n \int_{t_i}^{t_{i+1}} [f(t) - i\delta] dt = F(L) - \delta \sum_{i=1}^n (L - t_i) \quad (1)$$

Where $t = (t_1, t_2, \dots, t_n)$ is a vector from time to time to perform PM actions, the expected total cost to the lessor in the lease period includes minimum repair costs, penalty fees, and PM fees. As a result, the total expected is expressed as follows:

$$\begin{aligned} C(n, \delta, t) &= [Cm + C\tau G(\bar{\tau})]\Lambda + nCpm(\delta) \\ &= KF(L) + nCpm(\delta) - K\delta \sum_{i=1}^n (L - T_i) \end{aligned} \quad (2)$$

Where $K = Cm + C\tau \int_{\tau}^{\infty} G(t) dt$ is the expected cost for each fail. Meanwhile, without PM action ($n = 0$), the expected total cost reduces to $C_0 \equiv C(0, 0, \underline{t}; L) = KF(L)$.

$$C_0 \equiv C(0, 0, \underline{t}; L) = KF(L) \quad (3)$$

The purpose of the present research is to find the optimal PM policy (n^* , δ^* , t^*) for the lessor so that the total expected maintenance cost of Eq. (2) can be minimized. Note that there are $n + 2$ decision variables, including the number of PM actions (n), PM degrees (δ), and time period (t_i), in the destination function of Eq. (2). In the next section, the nature of the optimal PM policy is investigated and efficient algorithms are developed based on this trait.

For the calculation of equipment performance, OEE is used with the following equation (Supriatna, Singgih, Widodo, & Kurniati, 2017)

$$OEE = \left\{ \left(\frac{\text{loading time} - \text{down time}}{\text{loading time}} \right) \times \left(\frac{TCT \times PA}{OT} \right) \times \left(1 - \frac{\text{Process time}}{\text{loading time} - \text{down time}} \right) \right\} \quad (4)$$

Where TCT is the Theory of Cycle Time, PA is Process Amount, and OT is Operation Time.

Optimal Policy

Based on Eq. (2), it is obvious that there is a trade-off between $nCpm(\delta)$ and $K\delta \sum_{i=1}^n (L - t_i)$ in finding the optimal policy since $KF(L)$ is constant. Therefore, if $nCpm(\delta) - K\delta \sum_{i=1}^n (L - t_i) \geq 0$ for all $n > 0$, then preventive maintenance is not valuable, which means $n^* = 0$. In this case, the expected cost is $C_0 = KF(L)$. On the contrary, when if $nCpm(\delta) - K\delta \sum_{i=1}^n (L - t_i) < 0$ for all $n > 0$, n^* exists and the optimal policy is derived based on the following mathematical program:

$$\text{Minimize } C(n, \delta, \underline{t}) = C_0 + nCpm(\delta) - K\delta \sum_{i=1}^n (L - t_i)$$

Subject to

$$f(t_i) - i\delta \geq 0 \text{ for } i = 1, 2, 3, \dots, n \quad (5)$$

Since $f(t)$ is strictly increasing the function t , the reverse function of the failure rate, f^{-1} , also increases. Given that every $n > 0$ and $\delta > 0$, the following theorem shows the relationship between the optimal times of time t_i^* and the inverse function of the failure rate f^{-1} .

Theorem 1. If $h(t)$ is a strictly increasing function of t , then $t_i^* = f^{-1}(i\delta)$, Given any $n > 0$ and $\delta > 0$.

Theorem 1 shows that the optimal time do PM i is when $f(t_i) = i\delta$ and $\text{so } t_i^* = f^{-1}(i\delta)$. This result also shows that the failure rate must be reduced to zero after each PM. Using the results of Theorem 1, the objective function becomes $t_i^* = f(i\delta)$ as expressed as follows:

$$C(n, \delta | \underline{t}^*) = C_0 + nCpm(\delta) - K\delta \{nL - \sum_{i=1}^n [h^{-1}(i\delta)]\} \quad (6)$$

Now, there are only two decision variables, n and δ , in Eq. (6) to be determined. To find the optimal (n^* , δ^*), the case where n is given and the optimal level of maintenance δ_n^* must be determined previously. Then, the optimal number of PM actions can be obtained using the direct search method. Similarly, if $\sum_{i=1}^n [f^{-1}(i\delta)] \geq nL - \frac{nCpm(\delta)}{K\delta}$ for all $n > 0$, then $n^* = 0$ and it is expected that the cost generated $C_0 = KF(L)$. Therefore, in the following discussion, we will focus on the case where $\sum_{i=1}^n [f^{-1}(i\delta)] < nL - \frac{nCpm(\delta)}{K\delta}$. Given that each $n > 0$, the

following theorem shows that under some reasonable conditions, there is a unique $\delta_n^* \in \left[0, \frac{f(L)}{n}\right]$ which total expected cost is minimized.

Theorem 2. Given any $n > 0$, the following results can be clarified as follows:

- a) If $b - KL \geq 0$, then $\delta_n^* = 0$
- b) If $b - KL \geq 0$ and $2 \left(\frac{\partial \sum_{i=1}^n f^{-1}(i\delta)}{\partial \delta} \right) + \delta \left(\frac{\partial^2 \sum_{i=1}^n f^{-1}(i\delta)}{\partial \delta^2} \right) > 0$
 There exists a unique $\delta_n^* \in \left[0, \frac{f(L)}{n}\right]$, in which the expected total cost is minimized.

For a predetermined number of PM action $n > 0$, Theorem 2 shows that if the marginal cost of the PM action b is greater than the constant KL , then the optimal maintenance level is $\delta_n^* = 0$. In this case, the expected cost is $C_0 + na$, which implies that the PM action is not valuable and the optimal policy is $(n^*, \delta_n^*) = (0, 0)$. On the contrary, if the marginal cost is relatively low, then there is a unique optimal maintenance level when condition $2 \left(\frac{\partial \sum_{i=1}^n f^{-1}(i\delta)}{\partial \delta} \right) + \delta \left(\frac{\partial^2 \sum_{i=1}^n f^{-1}(i\delta)}{\partial \delta^2} \right) > 0$ satisfied. As we will show in the next section, this condition is reasonable because all Weibull distributions with increasing failure rates meet this condition.

Using the results of Theorem 2, the optimal level of maintenance can be easily obtained by the search method. Now, the final decision variable that is determined is the optimal amount of PM action in the lease period. In practice, there is a maximum number \bar{n} of PM actions that can be done within a limited lease period. Without general loss, it can be specified that $\bar{n} = \left\lfloor \frac{L}{\tau} \right\rfloor$ or a large number, to find the optimal value for n from 0 to \bar{n} . In short, the following algorithm can be used to find optimal policies and PM $(n^*, \delta^*, \underline{t}^*)$ for the lessor.

1. If $b - KL \geq 0$, then $(n^*, \delta^*, \underline{t}^*) = (0, 0, 0)$ and STOP.
2. Set $(n^*, \delta^*, \underline{t}^*) = (0, 0, 0)$, $C(n^*, \delta^*, \underline{t}^*) = C_0$, $\bar{n} = \left\lfloor \frac{L}{\tau} \right\rfloor$ and $n = 1$.
3. Search for $\delta_n^* \in \left[0, \frac{f(L)}{n}\right]$ such that $C(n, \delta_n^* | \underline{t}^*) = \min C(n, \delta | \underline{t}^*)$.
4. If $C(n, \delta_n^* | \underline{t}^*) < C(n^*, \delta^*, \underline{t}^*)$, then set $C(n^*, \delta^*, \underline{t}^*) = C(n, \delta_n^* | \underline{t}^*)$ and $(n^*, \delta^*, \underline{t}^*) = (n, \delta_n^*, \underline{t}^*)$.
5. If $n = \bar{n}$, count OEE then STOP; otherwise, set $n = n + 1$ and go to step 3.

Although there is an appropriate solution for the optimal level of δ_n^* , the nonlinear search in Step 3 may be time. In the next section, there is a solution for δ_n^* in Weibull's lifetime distribution. This result will significantly improve the efficiency of the algorithm above.

Weibull Case

The Weibull case is often discussed in the field of reliability because of the flexibility in the form of lifetime distribution. This section investigates the PM scheme proposed for the Weibull case in which a closed form solution that can be easily applied in practice is produced. There are two parameters for the Weibull distribution: the scale parameter $\alpha > 0$ and the shape parameter $\beta > 0$. The failure rate function of the Weibull distribution is $f(t) = \alpha\beta(\alpha t)^{\beta-1}$, then the failure rate increases in time (t). Likewise, it is assumed that the PM function costs $C_{pm}(\delta) = a + b\delta$, which increases linearly with the PM degree (δ). Then, from Eq. (5), the mathematical programs can be expressed as follows:

$$\begin{aligned} & \text{Minimize } C(n, \delta, \underline{t}) = K(\alpha L)^\beta + n(a + b\delta) - nK\delta L + K\delta \sum_{i=1}^n t_i \\ & \text{Subject to} \\ & \qquad f(t_i) - i\delta \geq 0 \text{ for all } i = 1, 2, \dots, n \end{aligned} \tag{7}$$

From the theorem 1, we know that $t_i^* = f^{-1}(i\delta)$. For the Weibull case, we have an inverse failure rate $t = f^{-1}(y) = wy^{\frac{1}{\beta-1}}$ for each $y > 0$, where $w = (\beta^{-1}\alpha^{-\beta})^{\frac{1}{\beta-1}}$. Therefore, $\sum_{i=1}^n [f^{-1}(i\delta)]$ (Eq. (6)) becomes: $w \left(\delta^{\frac{1}{\beta-1}} \right) \left(\sum_{i=1}^n i^{\frac{1}{\beta-1}} \right)$. Take the first derivative of $\sum_{i=1}^n [f^{-1}(i\delta)]$, with respect to δ , then the result will be:

$$\frac{\partial \sum_{i=1}^n [f^{-1}(i\delta)]}{\partial \delta} = w \left(\sum_{i=1}^n i^{\frac{1}{\beta-1}} \right) \left(\frac{1}{\beta-1} \right) \left(\delta^{\frac{1}{\beta-1}-1} \right) > 0 \quad (8)$$

For all $\delta > 0$, which means $\sum_{i=1}^n [f^{-1}(i\delta)]$ is an increase in function from δ . Next, the second derivative is $\sum_{i=1}^n [f^{-1}(i\delta)]$ with respect to δ as follows:

$$\frac{\partial^2 \sum_{i=1}^n [f^{-1}(i\delta)]}{\partial \delta^2} = \frac{\partial \sum_{i=1}^n [f^{-1}(i\delta)]}{\partial \delta} \left(\frac{2-\beta}{\beta-1} \right) \quad (9)$$

Using Eq. (h) and (i), condition $2 \left(\frac{\partial \sum_{i=1}^n f^{-1}(i\delta)}{\partial \delta} \right) + \delta \left(\frac{\partial^2 \sum_{i=1}^n f^{-1}(i\delta)}{\partial \delta^2} \right) > 0$ is given in theorem 2 to $\frac{\partial \sum_{i=1}^n [f^{-1}(i\delta)]}{\partial \delta} \left(\frac{2-\beta}{\beta-1} \right)$, which applies to the Weibull case with $\beta > 1$. Therefore, the following theorem can be applied to the Weibull case.

Theorem 3. For the Weibull case, given $n > 0$, the following results can be applied.

1. If $b - KL \geq 0$, then $\delta_n^* = 0$
2. If $b - KL < 0$, then $\delta_n^* = \left\{ n \left(L - \frac{b}{K} \right) \left(\frac{1}{w} \right) \left(\frac{1}{\left(\sum_{i=1}^n i^{\frac{1}{\beta-1}} \right)} \right) \left(\frac{\beta-1}{\beta} \right) \right\}^{(\beta-1)}$.

Theorem 3 shows that the optimal level of maintenance has a closed form solution for each $n > 0$. Therefore, Step 3 in the algorithm given in the previous section can be easily solved, and the optimal number of PM actions can be obtained by finding the optimal value n from 0 to \bar{n} . The above algorithm is reduced under the Weibull case as elucidated as follows:

1. If $b - KL \geq 0$, then $(n^*, \delta^*, \underline{t}^*) = (0, 0, 0)$ and STOP; otherwise set $n = 1$.
2. Set $\delta_n^* = \left\{ n \left(L - \frac{b}{K} \right) \left(\frac{1}{w} \right) \left(\frac{1}{\left(\sum_{i=1}^n i^{\frac{1}{\beta-1}} \right)} \right) \left(\frac{\beta-1}{\beta} \right) \right\}^{(\beta-1)}$ and $t_i^* = w(i\delta)^{\frac{1}{\beta-1}}$.
3. If $n = \bar{n}$ count OEE then STOP; otherwise, set $n = n + 1$ and go step 2.

NUMERICAL EXAMPLES

Consider that the equipment lifetime distribution is Weibull distributed with the scale parameter $\alpha > 0$ and the shape parameter $\beta > 1$, that is, $f(t) = \alpha\beta(\alpha t)^{\beta-1}$. To evaluate the optimal performance of PM policy implementation, the expected total cost without PM action (C_0), to be the baseline and specify $\Delta\% = \left((C_0 - C^*) / C_0 \right) 100$, which is the percentage of cost reduction, where C^* is the total expected cost based on optimal PM policy. Using this performance measure, the optimal PM policy under the proposed maintenance scheme is lowered and its performance is evaluated.

If the expected total cost for the lessor in each failure is $K = Cm + C\tau \int_{\tau}^{\infty} G(t) dt = 300$ and the repair time (T_m) follows Weibull (2.0.5). If the repair time exceeds $\tau = 2$, the lessor will be penalized. Subsequently, the PM

cost with the maintenance degree (δ) to be $C_{pm}(\delta) = 100 + 50\delta$. Then, Table 1 summarizes the numerical results for various combinations of β, L , and α .

TABLE 1. Numerical results

β	α	L	C_0	n^*	δ^*	C^*	$\Delta\%$
1.5	0.5	1	106.07	0	0	106.07	0
		2	300	1	0.41458	247.988	17.34
		3	551.14	1	0.51539	359.0819	34.85
		4	848.53	2	0.37914	467.1744	44.94
	1	1	300	1	0.79057	268.2384	10.59
		2	848.53	2	0.74162	504.6736	40.52
		3	1558.85	3	0.6748	711.6831	54.35
		4	2400	3	0.7849	894.725	62.72
2	0.5	1	75	0	0	75	0
		2	300	1	0.45833	273.9583	8.68
		3	675	2	0.47222	473.6111	29.84
		4	1200	2	0.63889	665.2778	44.56
	1	1	300	1	0.83333	295.8333	1.39
		2	1200	2	1.2222	727.7778	39.35
		3	2700	4	1.1333	1173.3333	56.54
		4	4800	6	1.0952	1621.4286	66.22

Table 1 summarizes the numerical results for various combinations of β, L , and α . For example, compilation $\alpha = 0.5, \beta = 2$, and $L = 4$, the total expected cost is $C_0 = 1200$, without the application of PM. However, under the optimal PM policy, $n^* = 2$, $\delta^* = 0.63889$, and the total expected cost becomes $C^* = 665.2778$. That is, in the lease period, 2 PM steps must be done at time $t_1^* = 1.2778$ and $t_2^* = 2.5556$. Whereas OEE after PM (t_1^*, t_2^*) is (98.79%, 99.27%). Under this PM policy, the estimated total costs can be reduced by 44.56%. If $\alpha = 0.5$, $\beta = 2$, and $L = 3$, the total expected cost is $C_0 = 675$, without PM step. However, under optimal PM policy, we have $n^* = 2$, $\delta^* = 0.47222$, and the total expected cost becomes $C^* = 473.6111$. PM is carried out at $t_1^* = 0.94444$ and $t_2^* = 1.8889$. Whereas OEE after PM (t_1^*, t_2^*) is (98.42%, 99.08%). Under this PM policy, the estimated total costs can be deferred to 29.84%.

Furthermore, from Table 1, we have the following observations:

1. When α and β increase, the optimal amount of PM action n^* increases, and the optimal degree of maintenance δ^* increases.
2. When the L lease period increases, it is expected that the total cost of C^* increases, the optimal number of PM actions n^* increases, and the percentage reduction in costs $\Delta\%$ also increases. These results indicate that PM actions have a significant impact on expected costs when the lease period is relatively long.
3. When δ^* increases, the equipment performance as measured by OEE increases.

CONCLUSION

Under the method of reducing the failure rate, this study proposes a maintenance scheme for equipment to be rented and derived from the optimal PM policy for leased equipment. Some structural properties of optimal policies are obtained, and efficient algorithms are developed based on these properties. Closed form solutions are obtained for cases where the age distribution of the equipment is Weibull. From the numerical examples of the Weibull case, we find that optimal policy performance with a maintenance degree is still better if the rent is a relatively long period. PM actions must be carried out in a leased period since the expected costs can be reduced significantly.

In the present study, the unique property of PM policy is optimally obtained for cases when lifetime distribution is a general one. However, for some generalizations of this maintenance scheme, uniqueness proposals may not be

suitable or the conditions of the existence of optimal policies may be complicated. Some possible generalizations, such as nonlinear maintenance costs, time dependent penalty fees, or various penalty schemes, can extend the problem for future studies in this area.

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